EXAMPLE ACI 318-08 RC-PN-001
Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

Figure 1: Flat Slab For Numerical Example
The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 150 pcf and an $f_c$ of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

**TECHNICAL FEATURES OF SAFE TESTED**

- Calculation of punching shear capacity, shear stress, and D/C ratio.

**RESULTS COMPARISON**

Table 1 shows the comparison of the SAFE punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

<table>
<thead>
<tr>
<th>Method</th>
<th>Shear Stress (ksi)</th>
<th>Shear Capacity (ksi)</th>
<th>D/C ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAFE</td>
<td>0.192</td>
<td>0.158</td>
<td>1.21</td>
</tr>
<tr>
<td>Calculated</td>
<td>0.193</td>
<td>0.158</td>
<td>1.22</td>
</tr>
</tbody>
</table>

**COMPUTER FILE:** ACI 318-08 RC-PN-001.FDB

**CONCLUSION**

The SAFE results show an exact comparison with the independent results.
**HAND CALCULATION**

Hand Calculation for Interior Column Using SAFE Method

\[ d = \frac{[(10 - 1) + (10 - 2)]}{2} = 8.5" \]

Refer to Figure 2.

\[ b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130" \]

**Figure 2: Interior Column, Grid B-2 in SAFE Model**

\[
\gamma_{r2} = 1 - \frac{1}{1 + \left( \frac{2}{3} \right) \sqrt{44.5}} = 0.4955
\]

\[
\gamma_{r3} = 1 - \frac{1}{1 + \left( \frac{2}{3} \right) \sqrt{\frac{20.5}{44.5}}} = 0.3115
\]

The coordinates of the center of the column \((x_1, y_1)\) are taken as \((0, 0)\).
The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>Side 1</th>
<th>Side 2</th>
<th>Side 3</th>
<th>Side 4</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>-10.25</td>
<td>0</td>
<td>10.25</td>
<td>0</td>
<td>N.A.</td>
</tr>
<tr>
<td>$y^2$</td>
<td>0</td>
<td>22.25</td>
<td>0</td>
<td>-22.25</td>
<td>N.A.</td>
</tr>
<tr>
<td>$L$</td>
<td>44.5</td>
<td>20.5</td>
<td>44.5</td>
<td>20.5</td>
<td>1105</td>
</tr>
<tr>
<td>$d$</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$Ld$</td>
<td>378.25</td>
<td>174.25</td>
<td>378.25</td>
<td>174.25</td>
<td>1105</td>
</tr>
<tr>
<td>$Ldx_2$</td>
<td>-3877.06</td>
<td>0</td>
<td>3877.06</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ldy_2$</td>
<td>0</td>
<td>3877.06</td>
<td>0</td>
<td>-3877.06</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0''
\]

\[
y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0''
\]

The following table is used to calculate $I_{XX}$, $I_{YY}$ and $I_{XY}$. The values for $I_{XX}$, $I_{YY}$ and $I_{XY}$ are given in the “Sum” column.

<table>
<thead>
<tr>
<th>Item</th>
<th>Side 1</th>
<th>Side 2</th>
<th>Side 3</th>
<th>Side 4</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>44.5</td>
<td>20.5</td>
<td>44.5</td>
<td>20.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$d$</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$x_2 - x_3$</td>
<td>-10.25</td>
<td>0</td>
<td>10.25</td>
<td>0</td>
<td>N.A.</td>
</tr>
<tr>
<td>$y_2 - y_3$</td>
<td>0</td>
<td>22.25</td>
<td>0</td>
<td>-22.25</td>
<td>N.A.</td>
</tr>
<tr>
<td>Parallel to</td>
<td>$Y$-Axis</td>
<td>$X$-axis</td>
<td>$Y$-Axis</td>
<td>$X$-axis</td>
<td>N.A.</td>
</tr>
<tr>
<td>Equations</td>
<td>$5b, 6b, 7$</td>
<td>$5a, 6a, 7$</td>
<td>$5b, 6b, 7$</td>
<td>$5a, 6a, 7$</td>
<td>N.A.</td>
</tr>
<tr>
<td>$I_{XX}$</td>
<td>64696.5</td>
<td>86264.6</td>
<td>64696.5</td>
<td>86264.6</td>
<td>301922.3</td>
</tr>
<tr>
<td>$I_{YY}$</td>
<td>39739.9</td>
<td>7151.5</td>
<td>39739.9</td>
<td>7151.5</td>
<td>93782.8</td>
</tr>
<tr>
<td>$I_{XY}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

From the SAFE output at Grid B-2:

\[ V_U = 189.45 \text{ k} \]

\[ \gamma_{12}M_{U2} = -156.39 \text{ k-in} \]

\[ \gamma_{13}M_{U3} = 91.538 \text{ k-in} \]
At the point labeled A in Figure 2, $x_4 = -10.25$ and $y_4 = 22.25$, thus:

$$
v_U = \frac{189.45}{130 \cdot 8.5} - \frac{156.39 \left[ 93782.8 \left( 22.25 - 0 \right) - (0) \left( -10.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2} - \frac{91.538 \left[ 301922.3 \left( -10.25 - 0 \right) - (0) \left( 22.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2}
$$

$$
v_U = 0.1714 - 0.0115 - 0.0100 = \mathbf{0.1499 \text{ ksi}} \text{ at point A}
$$

At the point labeled B in Figure 2, $x_4 = 10.25$ and $y_4 = 22.25$, thus:

$$
v_U = \frac{189.45}{130 \cdot 8.5} - \frac{156.39 \left[ 93782.8 \left( 22.25 - 0 \right) - (0) \left( 10.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2} - \frac{91.538 \left[ 301922.3 \left( 10.25 - 0 \right) - (0) \left( 22.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2}
$$

$$
v_U = 0.1714 - 0.0115 + 0.0100 = \mathbf{0.1699 \text{ ksi}} \text{ at point B}
$$

At the point labeled C in Figure 2, $x_4 = 10.25$ and $y_4 = -22.25$, thus:

$$
v_U = \frac{189.45}{130 \cdot 8.5} - \frac{156.39 \left[ 93782.8 \left( -22.25 - 0 \right) - (0) \left( 10.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2} - \frac{91.538 \left[ 301922.3 \left( 10.25 - 0 \right) - (0) \left( -22.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2}
$$

$$
v_U = 0.1714 + 0.0115 + 0.0100 = \mathbf{0.1930 \text{ ksi}} \text{ at point C}
$$

At the point labeled D in Figure 2, $x_4 = -10.25$ and $y_4 = -22.25$, thus:

$$
v_U = \frac{189.45}{130 \cdot 8.5} - \frac{156.39 \left[ 93782.8 \left( -22.25 - 0 \right) - (0) \left( -10.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2} - \frac{91.538 \left[ 301922.3 \left( -10.25 - 0 \right) - (0) \left( -22.25 - 0 \right) \right]}{(301922.3)(93782.8) - (0)^2}
$$

$$
v_U = 0.1714 + 0.0115 - 0.0100 = \mathbf{0.1729 \text{ ksi}} \text{ at point D}
$$
Point C has the largest absolute value of $v_u$, thus $v_{\text{max}} = 0.1930$ ksi

The shear capacity is calculated based on the smallest of ACI 318-08 equations 11-34, 11-35 and 11-36 with the $b_0$ and $d$ terms removed to convert force to stress.

$$\varphi_{vc} = \frac{0.75 \left(2 + \frac{4}{36/12}\right) \sqrt{4000}}{1000} = 0.158 \text{ ksi in accordance with equation 11-34}$$

$$\varphi_{vc} = \frac{0.75 \left(\frac{40 \cdot 8.5}{130} + 2\right) \sqrt{4000}}{1000} = 0.219 \text{ ksi in accordance with equation 11-35}$$

$$\varphi_{vc} = \frac{0.75 \cdot 4 \cdot \sqrt{4000}}{1000} = 0.190 \text{ ksi in accordance with equation 11-36}$$

Equation 11-34 yields the smallest value of $\varphi_{vc} = 0.158$ ksi and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_u}{\varphi_{vc}} = \frac{0.193}{0.158} = 1.22$$