EXAMPLE BS 8110-97 RC-BM-001
Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by BS 8110-97.
- The average shear stress in the beam is below the maximum shear stress allowed by BS 8110-97, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness ($1 \times 10^{20}$ kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20 and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined using the BS 8110-97 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the BS 8110-97 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.
Figure 1 The Model Beam for Flexural and Shear Design
GEOMETRY, PROPERTIES AND LOADING

Clear span \( l = 6000 \text{ mm} \)
Overall depth \( h = 500 \text{ mm} \)
Flange thickness \( d_s = 100 \text{ mm} \)
Width of web \( b_w = 300 \text{ mm} \)
Width of flange, \( b_f = 600 \text{ mm} \)
Depth of tensile reinf. \( d_c = 75 \text{ mm} \)
Effective depth \( d = 425 \text{ mm} \)
Depth of comp. reinf. \( d' = 75 \text{ mm} \)

Concrete strength \( f_c' = 30 \text{ MPa} \)
Yield strength of steel \( f_y = 460 \text{ MPa} \)
Concrete unit weight \( w_c = 0 \text{ kN/m}^3 \)
Modulus of elasticity \( E_c = 25 \times 10^5 \text{ MPa} \)
Modulus of elasticity \( E_s = 2 \times 10^8 \text{ MPa} \)
Poisson’s ratio \( v = 0.2 \)

Dead load \( P_d = 20 \text{ kN} \)
Live load \( P_l = 80 \text{ kN} \)

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 Also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

<table>
<thead>
<tr>
<th>Method</th>
<th>Moment (kN-m)</th>
<th>Reinforcement Area (sq-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAFE</td>
<td>312</td>
<td>20.90</td>
</tr>
<tr>
<td>Calculated</td>
<td>312</td>
<td>20.90</td>
</tr>
</tbody>
</table>

\( A_{s,\text{min}} = 195.00 \text{ sq-mm} \)
Table 2 Comparison of Shear Reinforcements

<table>
<thead>
<tr>
<th>Shear Force (kN)</th>
<th>Reinforcement Area, $\frac{A_x}{s}$ (sq-cm/m)</th>
<th>SAFE</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>156</td>
<td>6.50</td>
<td>6.50</td>
<td></td>
</tr>
</tbody>
</table>

**COMPUTER FILE:** BS 8110-97 RCBM-001.FDB

**CONCLUSION**

The SAFE results show an exact comparison with the independent results.
HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

\[ \gamma_{m, \text{steel}} = 1.15 \]
\[ \gamma_{m, \text{concrete}} = 1.50 \]
\[ A_{s, \text{min}} = 0.0013 b_w h \]
\[ = 195.00 \text{ sq-mm} \]

COMB80

\[ P = (1.4 P_d + 1.6 P_t) = 156 \text{ kN} \]
\[ M_* = \frac{N' l}{3} = 312 \text{ kN-m} \]

The depth of the compression block is given by:

\[ K = \frac{M}{f_{cu} b_f d^2} = 0.095963 < 0.156 \]

Then the moment arm is computed as:

\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d = 373.4254 \text{ mm} \]

The depth of the neutral axis is computed as:

\[ x = \frac{1}{0.45} (d - z) = 114.6102 \text{ mm} \]

And the depth of the compression block is given by:

\[ a = 0.9x = 103.1492 \text{ mm} > h_f \]

The ultimate resistance moment of the flange is given by:

\[ M_f = \frac{0.67 f_{cu} (b_f - b_w) h_f (d - 0.5 h_f)}{\gamma_c} = 150.75 \text{ kN-m} \]

The moment taken by the web is computed as:
and the normalized moment resisted by the web is given by:

\[ K_w = \frac{M_w}{f_{cu} b_w d^2} = 0.0991926 < 0.156 \]

If \( K_w \leq 0.156 \) (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right) \leq 0.95d = 371.3988 \text{ mm} \]

\[ A_s = \frac{M_f}{f_y (d - 0.5h_f)} + \frac{M_w}{f_{yc}} = 2090.4 \text{ sq-mm} \]

**Shear Design**

\[ v = \frac{V}{b_w d} \leq v_{\text{max}} = 1.2235 \text{ MPa} \]

\[ v_{\text{max}} = \min(0.8 \sqrt{f_{cu}}, 5 \text{ MPa}) = 4.38178 \text{ MPa} \]

The shear stress carried by the concrete, \( v_c \), is calculated as:

\[ v_c = \frac{0.79k_1 k_2}{\gamma_m} \left( \frac{100 A_s}{bd} \right)^{\frac{5}{6}} \left( \frac{400}{d} \right)^{\frac{1}{4}} = 0.3568 \text{ MPa} \]

\( k_1 \) is the enhancement factor for support compression, and is conservatively taken as 1.

\[ k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}} = 1.06266, \quad 1 \leq k_2 \leq \left( \frac{40}{25} \right)^{\frac{1}{3}} \]

\[ \gamma_m = 1.25 \]

\[ \frac{100 A_s}{bd} = 0.15 \]
\[ \left( \frac{400}{d} \right)^{1/4} = 1 \]

However, the following limitations also apply:

\[ 0.15 \leq \frac{100 A_s}{bd} \leq 3 \]

\[ \left( \frac{400}{d} \right)^{1/4} \geq 1 \]

\[ f_{cu} \leq 40 \text{ MPa (for calculation purposes only)} \] and \( A_s \) is the area of tension reinforcement.

Given \( v, v_c, \) and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows:

If \( v \leq (v_c + 0.4) \)

\[ \frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{yw}} \]

If \((v_c + 0.4) < v \leq v_{\text{max}}\)

\[ \frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yw}} \]

If \( v > v_{\text{max}}, \) a failure condition is declared.

**COMB80**

\[ P_d = 20 \text{ kN} \]
\[ P_l = 80 \text{ kN} \]
\[ V = 156 \text{ kN} \]

\[ \frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yw}} = 0.64967 \text{ sq-mm/mm} = 649.67 \text{ sq-mm/m} \]