TECHNICAL NOTE
PARAMETRIC P-M2-M3 HINGE MODEL

Overview

This technical note describes the Parametric P-M2-M3 hinge model, a non-linear frame hinge model that can couple the behavior in the axial and bending directions. Here P is the axial force, M2 the minor bending moment, and M3 the major bending moment.

The Parametric P-M2-M3 hinges use a P-M-M yield surface that is similar to that described in El-Tawil and Deierlein (2001). This document is intended to explain the fundamental concepts of the Parametric P-M2-M3 behavior. For details regarding the definition of the Parametric P-M2-M3 parameters, please see the Hinge Property Data form Help page in the program.

Two types of Parametric P-M2-M3 hinges are available: steel and concrete P-M2-M3 hinges. The two types of hinges differ only in the yield surface used. The Parametric Steel P-M2-M3 hinge is intended to model steel sections while the Parametric Concrete P-M2-M3 hinge is intended to model reinforced concrete sections.

Plasticity and Strain Hardening

The Parametric P-M2-M3 hinge uses plasticity theory to model P-M2-M3 interaction. This section discusses the general concepts for plasticity theory while subsequent sections will discuss specific behaviors of the Parametric P-M2-M3 hinge.

For biaxial stress, an elastic-plastic material has a yield surface, as shown in Figure 1. If the stress point is inside the yield surface the material is elastic. If the stress point is on the yield surface the material is yielded, and its behavior is elastic-plastic. Stress points outside the yield surface are not allowed.

The yield surface thus defines the strength of the material under biaxial stress. Plasticity theory defines the behavior of the material after it reaches the yield surface (i.e. after it yields). The ingredients of the theory are essentially as follows:

1. As long as the stress point stays on the yield surface, the material stays in a yielded state. However, the stress point does not remain in one place. The stresses can change after yield, even though the material is elastic-perfectly-plastic (e-p-p), which means that the stress point can move around the surface. The stress does not change after yield for an e-p-p material under for uniaxial stress, and hence biaxial stress is fundamentally different from uniaxial stress.

2. Point A in Figure 1 shows a yielded state defined by stresses σ_{1A} and σ_{1B}. Suppose that strain increments $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ are imposed, causing the stresses to change to σ_{2A} and σ_{2B} at point B. Plasticity theory says that some of the strain increment is an elastic increment and the remainder is plastic flow. The elastic part of the strain causes the change in stress. The plastic part causes no change in stress. This is why the behavior is referred to as elastic-plastic. For yield of an e-p-p material under uniaxial stress there is no stress change after yield. Hence, all of the strain after yield is plastic strain.

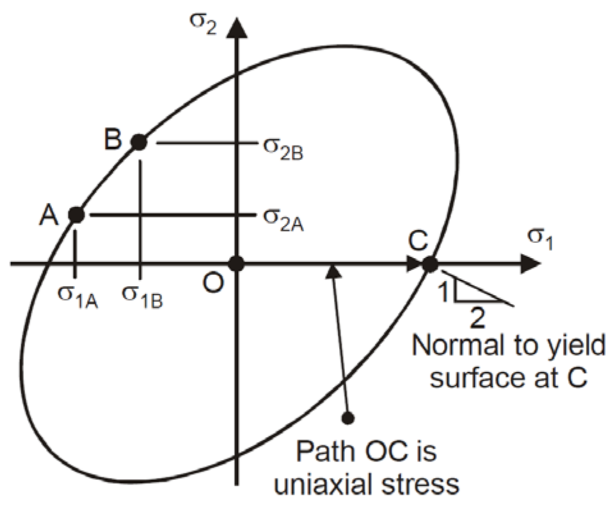

Figure 1 Some Features of the Yield Surface

3. Plasticity theory also defines the direction of plastic flow. That is, it defines the ratio between the 1-axis and 2-axis components of the plastic

strain. Essentially, the theory states that the direction of plastic flow is normal to the yield surface. For example, consider uniaxial stress along the 1-axis. As shown in Figure 1, the stress path is OC, and yield occurs at point C. After yield, the stress stays constant, and hence all subsequent strain is plastic. The normal to the yield surface at point C has 1-axis and 2-axis components in the ratio 2:1. Hence, the plastic strains are in this ratio, and the value of Poisson's ratio is 0.5 for plastic deformation.

The theory can be extended from the e-p-p case to the case with strain hardening. There are many hardening theories; the Parametric P-M2-M3 hinge uses the Mroz theory for strain hardening. For the case of trilinear behavior the Mroz theory is illustrated in Figure 2.

There are two yield surfaces, namely a Y surface (initial yield) and a larger U surface (ultimate strength). These surfaces both have the same shape. If the stress point is inside the Y surface the material is elastic. If the material is on the Y surface the material is elastic-plastic-strain-hardening. As the material hardens the Y surface moves, as indicated in the figure. When the stress point reaches the U surface, the material is elastic-plastic, as in the e-p-p case. Among other things, the Mroz theory specifies how the Y surface moves as the material strain hardens.

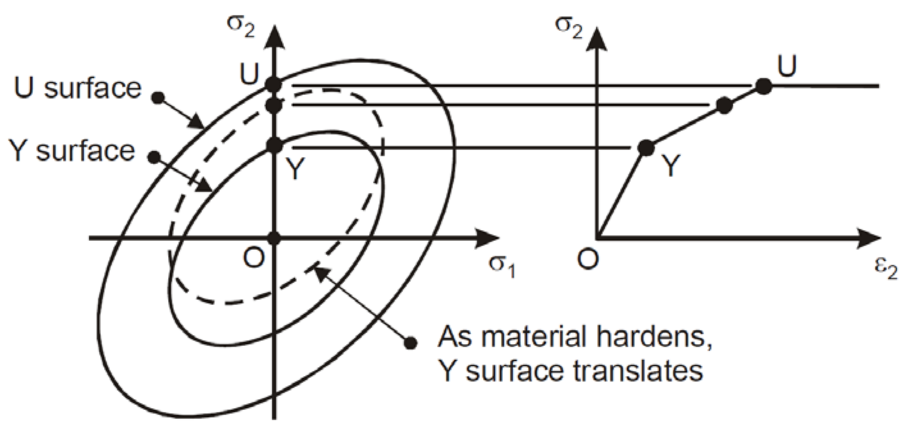

Figure 2 Trilinear Behavior with Mroz Theory

Extension to P-M2-M3 Interaction

Plasticity theory models the interaction between the material stresses σ_1 and σ_2. By analogy, plasticity theory can be extend to P-M2-M3 interaction in a column, where the axial force, P, and the bending moments, M2 and M3, interact with each other. For the e-p-p case, the yield surface is now the P-M strength interaction surface for the column section. Note that because plastic flow is normal to the yield surface, the P-M2-M3 hinge will generally extend or shorten axially as it yields in bending.

When used for steel sections, plasticity theory can correctly account for P-M interaction. Analyses of more complex cross sections show that plasticity theory can make reasonably accurate predictions of steel cross section behavior. Hence, inelastic hinges based on plasticity theory can be used to model steel columns with P-M interaction, for both pushover and dynamic earthquake analyses.

Plasticity theory does a mediocre job of modeling reinforced concrete behavior. The main error, especially for cyclic loading, is that for axial forces below the balance point, plasticity theory predicts plastic strain in tension after the yield surface is reached, for both bending directions. Hence, under cyclic bending the theory predicts that the column will progressively increase in length, possibly overestimating the axial growth for a reinforced concrete column. However, some growth is to be expected since the concrete resists compression but not tension.

It is suggested to consider Fiber P-M2-M3 hinges for cases where axial deformation is significant to the structural behavior. In contrast to plasticity-based hinges, Fiber P-M2-M3 hinges allow for more accurate calculation of the axial deformation because the uniaxial stress-strain relationships and hysteretic behavior of each of the individual fibers are considered. However, Fiber P-M2-M3 hinges may be computationally less efficient than plasticity-based hinges, especially when many fibers are used.

Yield Surface for a Parametric Steel P-M2-M3 Hinge

Figure 3 shows the yield surface for a steel section. The equations of the yield surface are essentially as follows.

In each P-M plane (P-M2 and P-M3):

$$f_{PM} = \left(\frac{P}{P_{Y0}}\right)^{\alpha} + \left(\frac{M}{M_{Y0}}\right)^{\beta} \quad (1)$$

where f_{PM} = yield function value = 1.0 for yielding, P = axial force, M = bending moment, P_{Y0} = yield force at M = 0 , and M_{Y0} = yield moment at P = 0.

Different values for the exponent α and the yield force P_{Y0} can be specified for tension and compression. Different values for the exponent α can also be used in the P-M2 and P-M3 planes.

For any value of P, Equation (1) defines the M values at which yield occurs, in both the P-M2 and P-M3 planes (put f_{PM} = 1 and solve for M). Call these values $M2_{YP}$ and $M3_{YP}$. The yield function in the M2-M3 plane is then:

$$f_{MM} = \left(\frac{M2}{M2_{YP}}\right)^{\gamma} + \left(\frac{M3}{M3_{YP}}\right)^{\beta} \quad (2)$$

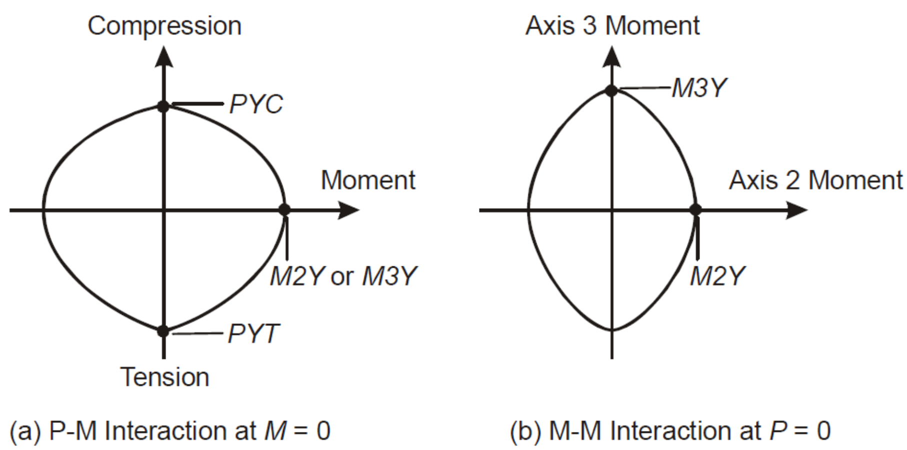

Figure 3 Yield Surface for a Parametric Steel P-M2-M3 Hinge

Yield Surface for a Parametric Concrete P-M2-M3 Hinge

Figure 4 shows the yield surface for a concrete section. The equations of the yield surface are essentially as follows.

In each P-M plane (P-M2 and P-M3):

$$f_{PM} = \left(\frac{P - P_B}{P_{Y0} - P_B}\right)^{\alpha} + \left(\frac{M}{M_{YB}}\right)^{\beta} \quad (3)$$

where f_{PM} = yield function value, = 1.0 for yield, P = axial force, P_B = axial force at balance point (assumed to be the same in both P-M planes), M = bending moment, P_{Y0} = yield force at M = 0 , and M_{Y0} = yield moment at P = P_B.

Different values for the exponent α and the yield force P_{Y0} can be specified for tension and compression. Different values for the exponent α can also be used in the P-M_2 and P-M_3 planes.

For any value of P, Equation (3) defines the M values at which yield occurs, in both the P-M2 and P-M3 planes (put f_{PM} = 1 and solve for M). The yield function in the M2-M3 plane is then given by Equation (2).

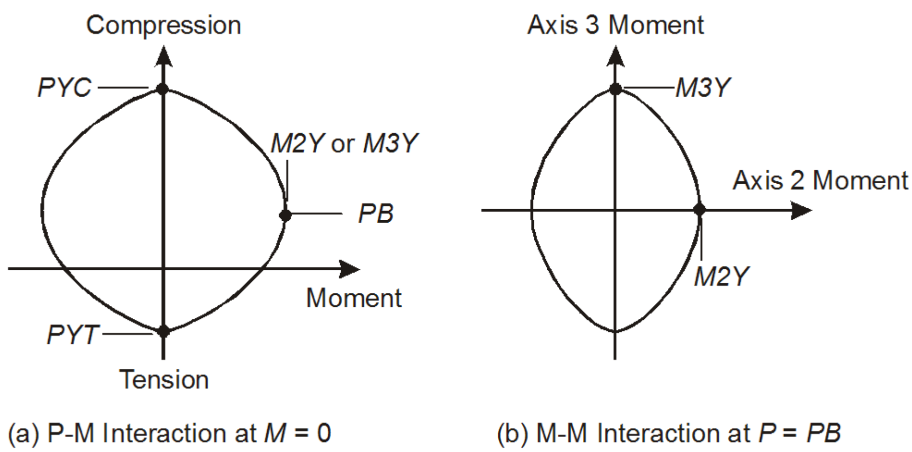

Figure 4 Yield Surface for a Parametric Concrete P-M2-M3 Hinge

Force-Deformation Behavior

Figure 5 shows the uniaxial force-deformation behavior of the Parametric P-M2-M3 hinge. The Parametric P-M2-M3 hinge behaves essentially rigid up to the yield point (the B point on the backbone curve). The behavior can be trilinear or elastic-perfectly-plastic – for trilinear behavior, the U point can be specified with a strength larger than that at the B point. The force is constant between the U and C points. Strength loss is optional and is controlled by the slope between the C and D points. The force is constant between points D and E, which extends indefinitely, representing residual strength in the hinge. The deformation at Points D and E cannot be explicitly specified in the axial direction because the onset of strength loss is determined based on bending deformations only.

The yield (Point B), ultimate (Point U), and residual (Point D) strengths can be independently specified for the following directions:

1. Compression, assuming $M_2 = M_3 = 0$
2. Tension, assuming $M_2 = M_3 = 0$
3. Bending moment around axis 2, M_2, assuming $M_3 = 0$ and $P = 0$ for the steel hinge or $P = P_B$ for the concrete hinge.
4. Bending moment around axis 3, M_3, assuming $M_2 = 0$ and $P = 0$ for the steel hinge or $P = P_B$ for the concrete hinge.

The yield surfaces for P-M2-M3 interaction of this hinge only allows for doubly-symmetrical cross sections with equal positive and negative bending strengths each for M2 and M3.

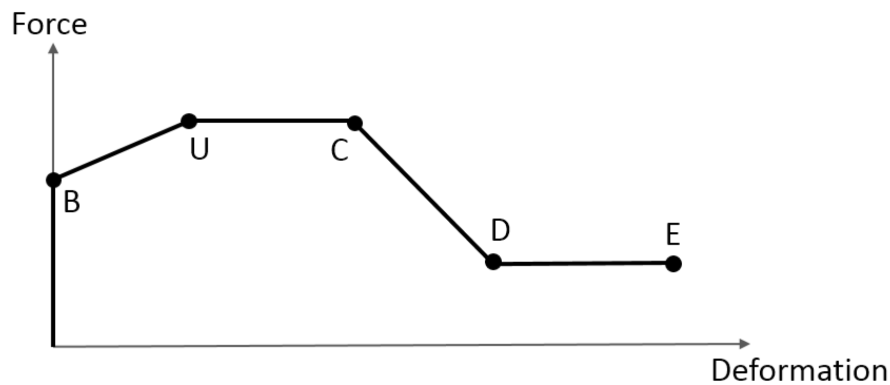

Figure 5 Uniaxial Behavior

Strength Loss

For the onset of strength loss (the C point on the backbone curve), the P-M2-M3 hinges uses bending deformations only (i.e., axial deformations are not considered).

When you specify the C point for strength loss you must specify C point bending deformations about both Axis 2 and Axis 3. The C point is reached when the following equation is satisfied:

$$\left(\frac{D2}{D2C}\right)^2 + \left(\frac{D3}{D3C}\right)^2 = 1 \qquad (4)$$

where *D2*, *D3* are the current bending deformations about Axes 2 and 3, and *D2C*, *D3C* are the C point deformations.

You must also specify the ratio between the C point strength and the D point strength. You can specify one ratio for bending moment and a different ratio for axial force. If you specify the same ratio for axial force as for bending, as the hinge loses strength the yield surface decreases in size without changing shape. If you specify different ratios, the yield surface reduces in size and changes shape.

When you specify the E point must specify E-point bending deformations about both Axis 2 and Axis 3, and also an E-point axial deformation. These bending and axial deformations are checked separately. The E point is reached when either deformation exceeds the corresponding E-point deformation, whichever occurs first.

The E point in bending is reached when the following equation is satisfied:

$$\left(\frac{D2}{D2E}\right)^2 + \left(\frac{D3}{D3E}\right)^2 = 1 \qquad (5)$$

where *D2*, *D3* are the current bending deformations about Axes 2 and 3, and *D2E*, *D3E* are the E point deformations.

After strength loss occurs for trilinear behavior, the behavior is assumed to become elastic-perfectly-plastic.

Energy Degradation

This section describes the behavior of energy degradation on the hysteretic shape of the Parametric P-M2-M3 hinge. Figure 6 shows a loop for the e-p-p case with energy degradation for the uniaxial case. The axial and bending stiffnesses are both reduced, in the same proportion.

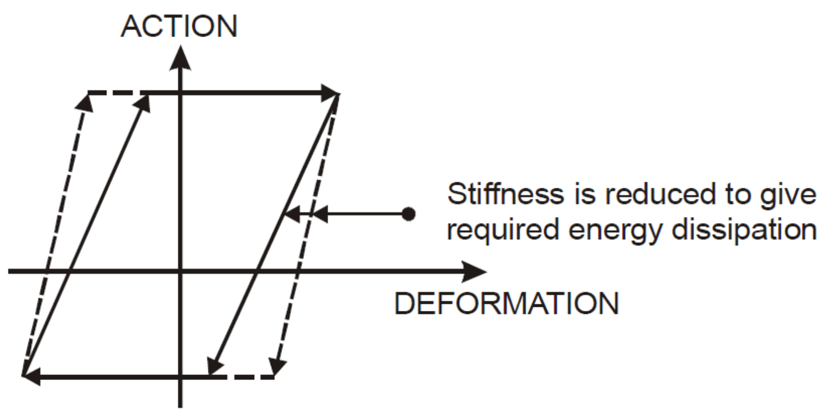

Figure 6 Degraded Loop for E-P-P Behavior

The following method is used to set the loop properties:

1. As part of the hinge definition, a relationship can be specified between the deformation of the hinge and the corresponding energy degradation factor. This factor is the area of the degraded hysteresis loop divided by the area of the non-degraded loop.

2. For the current state of the hinge, e_{pos} and e_{neg} are the energy degradation factors at the maximum positive and negative deformations. Note that these are the maximum deformations up to the current point in the analysis, not necessarily the deformations at the limits of the current deformation cycle.

3. The energy degradation factor, e, for the loop as a whole is the larger of e_{pos} and e_{neg}. The degraded unloading stiffness is calculated to make the area of the degraded loop equal to e times the area of the non-degraded loop.

The behavior of energy degradation for the trilinear case is shown in Figure 7. Figure 7(a) shows the case where the positive and negative deformation are both smaller than the U point deformation. The energy dissipation factor, e, is calculated as for the e-p-p case. The hardening stiffness is kept constant and only the elastic stiffness is reduced for the unloading branch – this results in an increase in the elastic deformation range and the elastic force range.

Figure 7(b) shows the case where the positive and negative deformations are both larger than the U point deformation. The energy degradation in this case is a combination of that shown in Figure 6 and 7(a).

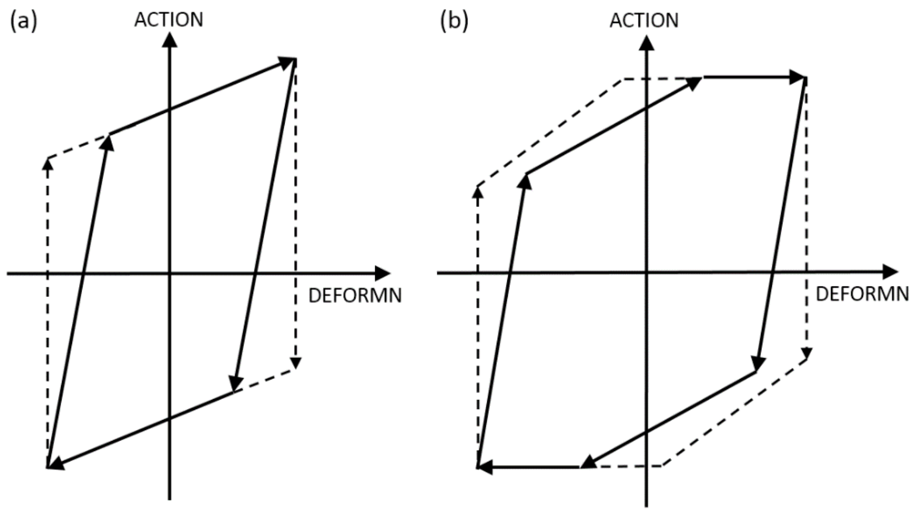

Figure 7 Degraded Loop for Trilinear Behavior: (a) Before U Point, (b) After U Point

Deformation Capacity Ratios

For deformation demand-capacity ratios, the Parametric P-M2-M3 hinge uses bending deformations only.

When you specify the deformation capacities, you can specify bending deformation capacities for up to 3 performance levels (called IO, LS, and CP). The deformation demand-capacity ratio is calculated as follows:

$$D/C\ Ratio = \sqrt{\left(\frac{D2}{D2_{cap}}\right)^2 + \left(\frac{D3}{D3_{cap}}\right)^2} \tag{6}$$

where *D2*, *D3* are the current bending deformations about Axes 2 and 3, and $D2_{cap}$, $D3_{cap}$ are the deformation capacities at a given performance level.

For steel P-M2-M3 hinges you can specify that the deformation capacities depend on the axial force. For a concrete P-M2-M3 hinge you can specify that the deformation capacities depend on both the axial force and the shear force.

References

El-Tawil, S. and Deierlein, G. "Nonlinear Analysis of Mixed Steel-Concrete Frames, I: Element Formulation." *Journal of Structural Engineering*, Vol. 126, No. 6, June 2001.

El-Tawil, S. and Deierlein, G. "Nonlinear Analysis of Mixed Steel-Concrete Frames, II: Implementation and Verification." *Journal of Structural Engineering*, Vol. 126, No. 6, June 2001.