TECHNICAL NOTE
MATERIAL STRESS-STRAIN CURVES

General

All material types have stress-strain curves that are defined by a series of user-specified stress-strain points. In addition, concrete, rebar and structural steel and tendon materials have several special types of parametric stress-strain curve definitions. For concrete, Simple and Mander parametric definitions are available. For rebar, Simple and Park parametric definitions are available. For structural steel, a Simple parametric definition is available. For tendons, a 250Ksi strand and a 250Ksi strand definition are available.

User Stress-Strain Curves

User stress-strain curves apply to all material types. They are defined by a series of stress-strain points (ε, f). One of the stress-strain points must be at (0,0). User stress-strain curves may be input and viewed as standard stress-strain curves or as normalized curves. Normalized curves plot f/f_y versus $\varepsilon/\varepsilon_y$, where $\varepsilon_y = f_y/E$. The program stores user stress-strain curves as normalized curves. Thus, if the E or f_y value for a material is changed, the stress-strain curve for that material automatically changes.

Rebar Parametric Stress-Strain Curves

Two types of parametric stress-strain curves are available for rebar. They are Simple and Park. The two are identical, except in the strain hardening region where the Simple curves use a parabolic shape and the Park curves use an empirical shape. The following parameters define the rebar parametric stress-strain curves:

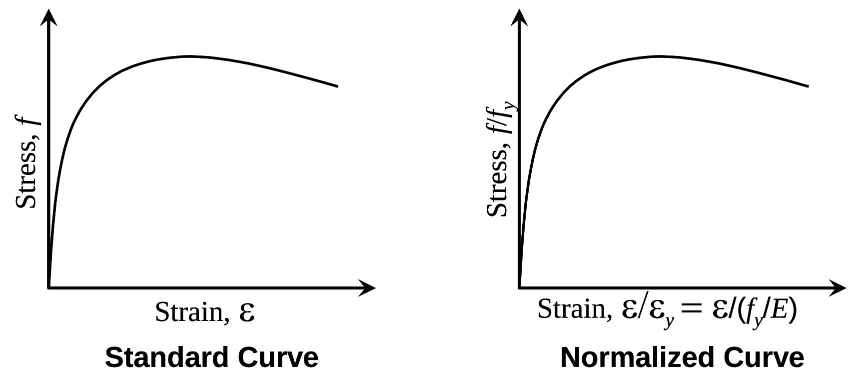

Figure 1 Stress-Strain Curves

ε = Rebar strain

f = Rebar stress

E = Modulus of elasticity

f_y = Rebar yield stress

f_u = Rebar ultimate stress capacity

ε_{sh} = Strain in rebar at the onset of strain hardening

ε_u = Rebar ultimate strain capacity

The rebar yield strain, ε_y, is determined from $\varepsilon_y = f_y /E$.

The stress-strain curve has three regions. They are an elastic region, a perfectly plastic region, and a strain hardening region. Different equations are used to define the stress-strain curves in each region.

The rebar parametric stress-strain curves are defined by the following equations:

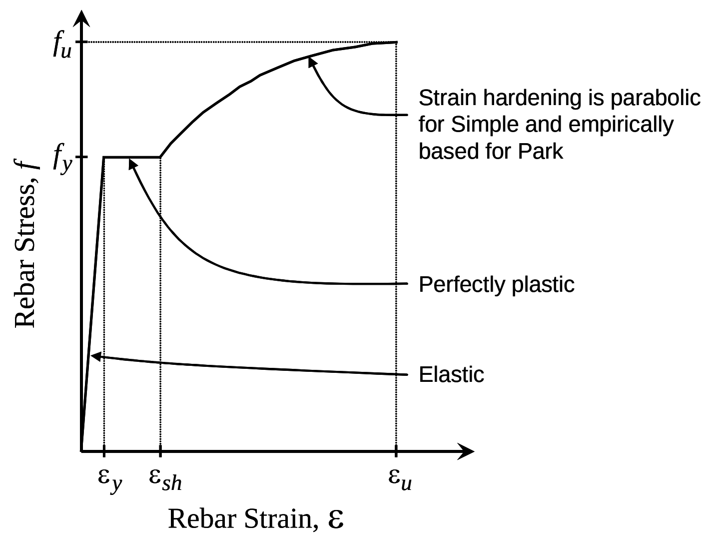

Figure 2 Rebar Parametric Stress-Strain Curve

For $\varepsilon \leq \varepsilon_y$ (elastic region)

$$f = E\varepsilon$$

For $\varepsilon_y < \varepsilon \leq \varepsilon_{sh}$ (perfectly plastic region)

$$f = f_y$$

For $\varepsilon_{sh} < \varepsilon \leq \varepsilon_u$ (strain hardening region)

For Simple parametric curves,

$$f = f_y + \left(f_u - f_y\right)\sqrt{\frac{\varepsilon - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}}$$

For Park parametric curves,

$$f = f_y\left(\frac{m\left(\varepsilon - \varepsilon_{sh}\right) + 2}{60\left(\varepsilon - \varepsilon_{sh}\right) + 2} + \frac{\left(\varepsilon - \varepsilon_{sh}\right)\left(60 - m\right)}{2\left(30r + 1\right)^2}\right)$$

where,

$$r = \varepsilon_u - \varepsilon_{sh}$$

$$m = \frac{(f_u / f_y)(30r + 1)^2 - 60r - 1}{15r^2}$$

Both the Simple and the Park parametric stress-strain curves have the option to use Caltrans default strain values for the curves. Those default values are dependent on rebar size.

With A_s denoting the area of a rebar, the Caltrans default strains used by the program are as follows:

ε_u = 0.090 for $A_s \leq 1.40$ in^2

ε_u = 0.060 for $A_s > 1.40$ in^2

ε_{sh} = 0.0150 for $A_s \leq 0.85$ in^2

ε_{sh} = 0.0125 for $0.85 < A_s \leq 1.15$ in^2

ε_{sh} = 0.0115 for $1.15 < A_s \leq 1.80$ in^2

ε_{sh} = 0.0075 for $1.80 < A_s \leq 3.00$ in^2

ε_{sh} = 0.0050 for $A_s > 3.00$ in^2

In terms of typical bar sizes, the default values are as follows:

ε_u = 0.090 for #10 (#32m) bars and smaller

ε_u = 0.060 for #11 (#36m) bars and larger

ε_{sh} = 0.0150 for #8 (#25m) bars

ε_{sh} = 0.0125 for #9 (#29m) bars

ε_{sh} = 0.0115 for #10 and #11 (#32m and #36m) bars

ε_{sh} = 0.0075 for #14 (#43m) bars

ε_{sh} = 0.0050 for #18 (#57m) bars

Simple Structural Steel Parametric Stress-Strain Curve

The Simple structural steel parametric stress-strain curve has four distinct regions. They are an elastic region, a perfectly plastic region, a strain hardening region, and a softening region.

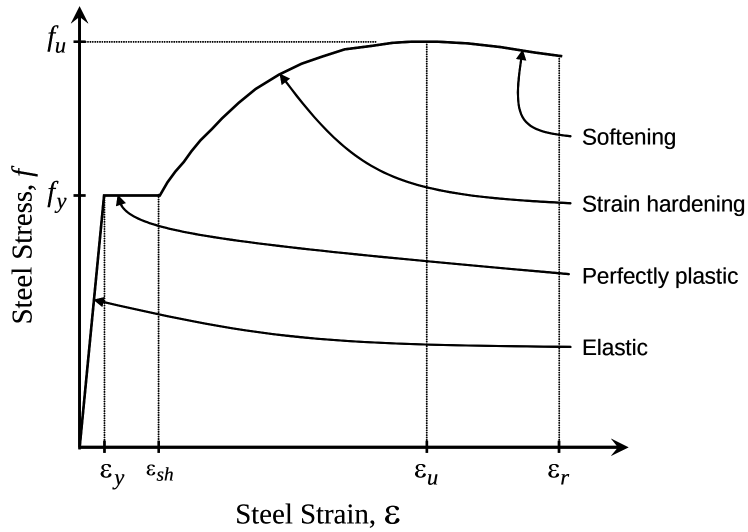

Figure 3 Simple Structural Steel Parametric Stress-Strain Curve

The following parameters define the structural steel Simple stress-strain curve.

ε = Steel strain

f = Steel stress

E = Modulus of elasticity

f_y = Steel yield stress

f_u = Steel maximum stress

ε_{sh} = Strain at onset of strain hardening

ε_u = Strain corresponding to steel maximum stress

ε_r = Strain at steel rupture

The steel yield strain, ε_y, is determined from $\varepsilon_y = f_y / E$.

The structural steel Simple parametric stress-strain curve is defined by the following equations:

For $\varepsilon \le \varepsilon_y$ (elastic region),

$$f = E\varepsilon$$

For $\varepsilon_y < \varepsilon \leq \varepsilon_{sh}$ (perfectly plastic region),

$$f = f_y$$

For $\varepsilon_{sh} < \varepsilon \leq \varepsilon_r$ (strain hardening and softening regions),

$$f = f_y\left(1 + r\left(\frac{f_u}{f_y} - 1\right)e^{(1-r)}\right)$$

where,

$$r = \frac{\varepsilon - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}$$

The strain hardening and softening expression is from Holzer et al. (1975).

Tendon 250Ksi Strand Stress-Strain Curve

The following parameters define the 250Ksi Strand stress-strain curve.

f = Tendon stress

ε = Tendon strain

E = Modulus of elasticity

ε_y = Tendon yield stress

ε_u = Tendon ultimate strain

The tendon ultimate strain, ε_u, is taken as 0.03. The tendon yield strain, ε_y, is determined by solving the following quadratic equation, where E is in ksi. The larger obtained value of ε_y is used.

$$E\varepsilon_y{}^2 - 250\varepsilon_y + 0.25 = 0$$

The stress-strain curve is defined by the following equations:

For $\varepsilon \leq \varepsilon_y$,

$$f = E\varepsilon$$

For $\varepsilon_y < \varepsilon \leq \varepsilon_u$

$$f = 250 - \frac{0.25}{\varepsilon}$$

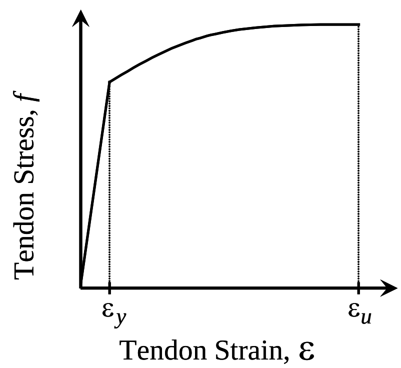

Figure 4 Tendon 250Ksi Strand Stress-Strain Curve

Tendon 270Ksi Strand Stress-Strain Curve

The following parameters define the 270Kksi Strand stress-strain curve.

f = Tendon stress

ε = Tendon strain

E = Modulus of elasticity

ε_y = Tendon yield stress

ε_u = Tendon ultimate strain

The tendon ultimate strain, ε_u, is taken as 0.03. The tendon yield strain, ε_y, is determined by solving the following quadratic equation, where E is in ksi. The larger obtained value of ε_y is used.

$$E\varepsilon_y^2 - (270 + 0.007E)\varepsilon_y + 1.93 = 0$$

The stress-strain curve is defined by the following equations:

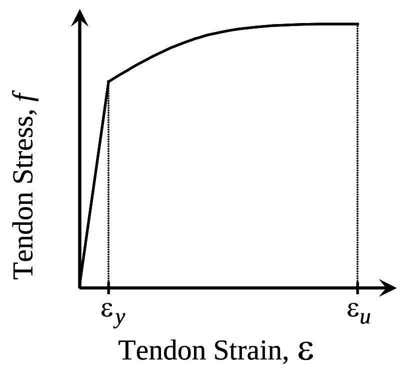

Figure 5 Tendon 270Ksi Strand Stress-Strain Curve

For $\varepsilon \leq \varepsilon_y$,

$$f = E\varepsilon$$

For $\varepsilon_y < \varepsilon \leq \varepsilon_u$

$$f = 270 - \frac{0.04}{\varepsilon - 0.007}$$

Simple Concrete Parametric Stress-Strain Curve

The compression portion of the Simple concrete parametric stress-strain curve consists of a parabolic portion and a linear portion. The following parameters define the Simple concrete parametric stress-strain curve.

ε = Concrete strain

f = Concrete stress

f'_c = Concrete compressive strength

ε'_c = Concrete strain at f'_c

ε_u = Ultimate concrete strain capacity

The concrete Simple parametric stress-strain curve is defined by the following equations:

For $\varepsilon \leq \varepsilon'_c$ (parabolic portion),

$$f = f'_c\left\{2\left(\frac{\varepsilon}{\varepsilon'_c}\right) - \left(\frac{\varepsilon}{\varepsilon'_c}\right)^2\right\}$$

For $\varepsilon'_c < \varepsilon \leq \varepsilon_u$ (linear portion),

$$f = f'_c\left\{1 - 0.2\left(\frac{\varepsilon - \varepsilon'_c}{\varepsilon_u - \varepsilon'_c}\right)\right\}$$

The tensile yield stress for the Simple concrete curve is taken at $7.5\sqrt{f'_c}$ in psi.

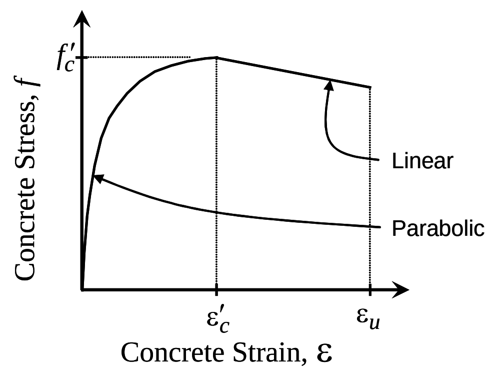

Figure 6 Simple Concrete Parametric Stress-Strain Curve

Mander Concrete Parametric Stress-Strain Curve

The Mander concrete stress-strain curve is documented in the following reference:

> Mander, J.B., M.J.N. Priestley, and R. Park 1984. *Theoretical Stress-Strain Model for Confined Concrete*. Journal of Structural Engineering. ASCE. 114(3). 1804-1826.

The Mander concrete stress-strain curve calculates the compressive strength and ultimate strain values as a function of the confinement (transverse reinforcing) steel. The following types of Mander stress-strain curves are possible.

- Mander – Unconfined Concrete

- Mander – Confined Concrete – Rectangular Section
- Mander – Confined Concrete – Circular Section

The Mander unconfined concrete stress-strain curve can be generated from material property data alone. The Mander confined concrete stress-strain curves requir both material property data and section property data. The following section frame section types have appropriate section property data for Mander confined concrete:

- Rectangular Section
- Circular Section

The following section objects in Section Designer sections have appropriate property data for Mander confined concrete:

- Solid Rectangle
- Solid Circle
- Poly
- Caltrans Hexagon
- Caltrans Octagon
- Caltrans Round
- Caltrans Square

When a material with Mander stress-strain curves is assigned to a section that has appropriate section property data for Mander confined concrete, the type of Mander stress-strain curve used for that section is determined from the section property data. When the section does not have appropriate data for Mander confined concrete, the Mander unconfined concrete stress-strain curve is always used.

Mander Unconfined Concrete Stress-Strain Curve

The compression portion of the Mander unconfined stress-strain curve consists of a curved portion and a linear portion. The following parameters define the Mander unconfined concrete stress-strain curve.

ε = Concrete strain

f = Concrete stress

E = Modulus of elasticity

f'_c = Concrete compressive strength

ε'_c = Concrete strain at f'_c

ε_u = Ultimate concrete strain capacity

The Mander unconfined concrete stress-strain curve is defined by the following equations:

For $\varepsilon \leq 2\varepsilon'_c$ (curved portion),

$$f = \frac{f'_c x r}{r - 1 + x^r}$$

where

$$x = \varepsilon / \varepsilon'_c$$

$$r = \frac{E}{E - \left(f'_c / \varepsilon'_c\right)}$$

For $2\varepsilon'_c < \varepsilon \leq \varepsilon_u$ (linear portion),

$$f = \left(\frac{2 f'_c r}{r - 1 + 2^r}\right)\left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - 2\varepsilon'_c}\right)$$

where r is as defined previously for the curved portion of the curve.

The tensile yield stress for the Mander unconfined curve is taken at $7.5\sqrt{f'_c}$ in psi.

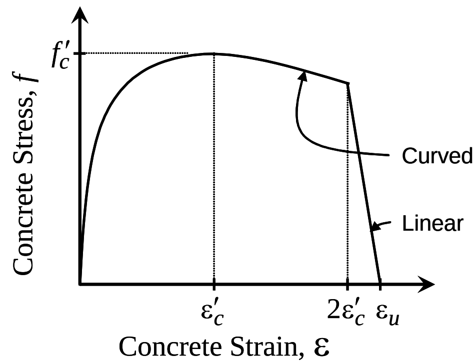

Figure 7 Mander Unconfined Concrete Stress-Strain Curve

Mander Confined Concrete Stress-Strain Curve

For the compression portion of the Mander confined concrete stress-strain curves, the compressive strength and the ultimate strain of the confined concrete are based on the confinement (transverse reinforcing) steel. The following parameters define the Mander confined concrete stress-strain curve:

ε = Concrete strain

f = Concrete stress

E = Modulus of elasticity (tangent modulus)

E_{sec} = Secant modulus of elasticity

f'_c = Compressive strength of unconfined concrete

f'_{cc} = Compressive strength of confined concrete; this item is dependent on the confinement steel provided in the section and is explained later

ε'_c = Concrete strain at f'_c

ε_u = Ultimate concrete strain capacity for unconfined concrete and concrete spalling strain for confined concrete

ε'_{cc} = Concrete strain at f'_{cc}

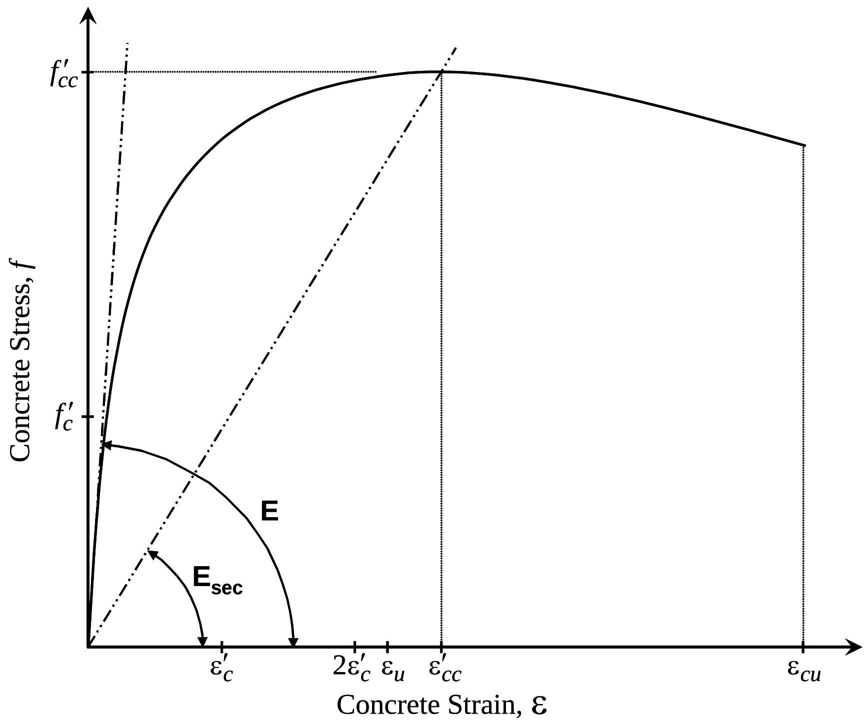

Figure 8 Mander Confined Concrete Stress-Strain Curve

ε_{cu} = Ultimate concrete strain capacity for confined concrete; this item is dependent on the confined steel provided in the section and is explained later

The Mander confined concrete stress-strain curve is defined by the following equations:

$$f = \frac{f'_{cc} x r}{r - 1 + x^r}$$

where,

$$\varepsilon'_{cc} = \left\{5\left(\frac{f'_{cc}}{f'_c} - 1\right) + 1\right\}\varepsilon'_c$$

$$x = \varepsilon / \varepsilon'_{cc}$$

$E_{sec} = f'_{cc} / \varepsilon'_{cc}$

$r = E / (E - E_{sec})$

Mander Confined Concrete Compressive Strength, f'_{cc}

The following parameters are used in the explanation of f'_{cc} :

A_c = Area of concrete core measured from centerline to centerline of confinement steel

A_{cc} = Concrete core area excluding longitudinal bars; $A_{cc} = A_c\ (1\text{-}\rho_{cc})$

A_e = Concrete area that is effectively confined

A_{sc} = Area of a circular hoop or spiral confinement bar

A_{sL} = Total area of all longitudinal bars

A_{sx} = Area of rectangular hoop legs extending in the x-direction

A_{sy} = Area of rectangular hoop legs extending in the y-direction

b_c = Centerline to centerline distance between rectangular perimeter hoop legs that extend in the y-direction

d_c = Centerline to centerline distance between rectangular perimeter hoop legs that extend in the x-direction

d_s = Diameter of circular hoops or spirals of confinement steel measured from centerline to centerline of steel

f'_c = Unconfined concrete compressive strength

f_L = Lateral pressure on confined concrete provided by the confinement steel

f'_L = Effective lateral pressure on confined concrete provided by the confinement steel

f_{yh} = Yield stress of confinement steel

K_e = Coefficient measuring the effectiveness of the confinement steel

s = Centerline to centerline longitudinal distance between hoops or spirals

s' = Clear longitudinal distance between hoops or spirals

w' = Clear transverse distance between adjacent longitudinal bars with cross ties

ρ_{cc} = Longitudinal steel ratio; $\rho_{cc} = A_{sL}/A_c$

ρ_s = Volumetric ratio of transverse confinement steel to the concrete core

ρ_x = Steel ratio for rectangular hoop legs extending in the x-direction; $\rho_x = A_{sx}/sd_c$

ρ_y = Steel ratio for rectangular hoop legs extending in the y-direction; $\rho_y = A_{sy}/sb_c$

For circular cores:

$$\rho_s = \frac{4A_{sc}}{d_s s}$$

$$f_L = \frac{\rho_s f_{yh}}{2}$$

$$A_{cc} = \frac{\pi}{4} d_s^2 (1 - \rho_{cc})$$

$$A_e = \frac{\pi}{4}\left(d_s - \frac{s'}{2}\right)^2 \text{ for tied hoops}$$

$$A_e = \frac{\pi}{4} d_s \left(d_s - \frac{s'}{2}\right) \text{ for spirals}$$

$$K_e = \frac{A_e}{A_{cc}}$$

$$f'_L = K_e f_L$$

$$f'_{cc} = f'_c \left(2.254\sqrt{1 + \frac{7.94 f'_L}{f'_c}} - 2\frac{f'_L}{f'_c} - 1.254\right)$$

For rectangular cores

$$\rho_x = \frac{A_{sx}}{sd_c}$$

$$\rho_y = \frac{A_{sy}}{sb_c}$$

$$f_{Lx} = \rho_x f_{yh}$$

$$f_{Ly} = \rho_y f_{yh}$$

$$A_e = \left(b_c d_c - \sum_{i-1}^{n} \frac{\left(w_i'\right)^2}{6} \right)\left(1 - \frac{s'}{2b_c}\right)\left(1 - \frac{s'}{2d_c}\right)$$

$$A_{cc} = b_c d_c$$

$$K_e = \frac{A_e}{A_{cc}}$$

$$f_{Lx}' = K_e f_{Lx}$$

$$f_{Ly}' = K_e f_{Ly}$$

After f_{Lx}' and f_{Ly}' are known, f_{cc}' is determined using a chart for the multiaxial failure criterion in terms of two lateral confining stresses that is published in the previously referenced article, Mander et al. (1984).

Mander Confined Concrete Ultimate Strain Capacity, ε_{cu}

The Mander confined concrete ultimate strain capacity, ε_{cu}, is a function of the confinement steel. The following figure shows the Mander stress-strain curves for confined and unconfined concrete. The difference between the confined and unconfined curves is shown shaded.

The shaded area shown in Figure 9 represents the additional capacity provided by the confinement steel for storing strain energy.

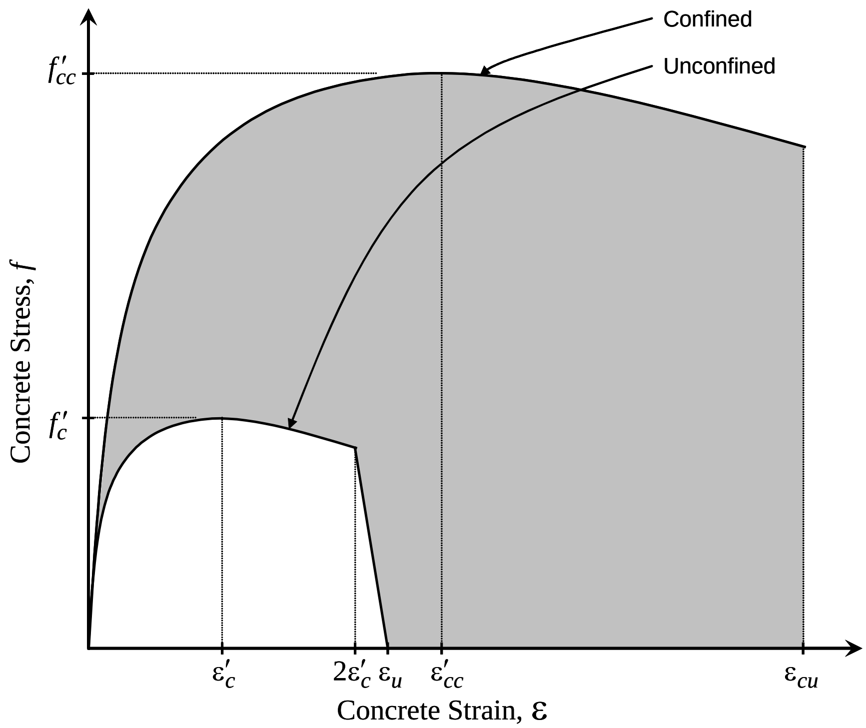

Figure 9 Mander Confined and Unconfined Stress-Strain Curves

This area is limited to the energy capacity available in the area under the confinement steel stress-strain curve up to the ultimate steel strain, ε_u.

Suppose A_1 is the shaded area between the Mander confined and unconfined curves and A_2 is the area under the confinement steel stress-strain curve. Further suppose ρ_s is the volumetric ratio of confinement steel to the concrete core. Then, equating energies under the concrete and confinement steel stress-strain curves gives:

$$A_1 = \rho_S a_2$$

The program determines the appropriate value of the confined concrete ultimate straining, ε_{cu}, by trial and error, equating energies as described previously. When the $A_1 = \rho_s A_2$ relationship is satisfied, the correct value of ε'_{cu} has been found.

The tensile yield stress for the Mander confined curves is taken as $7.5\sqrt{f_c'}$ in psi.

References

Holzer et al. 1975. SINDER. *A Computer Code for General Analysis of Two-Dimensional Reinforced Concrete Structures.* Report. AFWL-TR-74-228 Vol. 1. Air Force Weapons Laboratory, Kirtland, AFB, New Mexico.

Mander, J.B., M.J.N. Priestley, and R. Park 1984. *Theoretical Stress-Strain Model for Confined Concrete*. Journal of Structural Engineering. ASCE. 114(3). 1804-1826.