TECHNICAL NOTE
MODIFIED DARWIN-PECKNOLD 2-D REINFORCED CONCRETE MATERIAL MODEL

Overview

This technical note describes the Modified Darwin-Pecknold reinforced concrete material model, a two-dimensional concrete material model that can account directly for the interaction between bending and shear in shear wall structures.

In an actual wall, especially in a "squat" wall, there can be substantial coupling between axial-bending and shear. In particular, the shear strength of a wall may depend substantially on the axial forces and bending moments. The 2D concrete model attempts to model this coupling directly.

The model is a co-axially rotating smeared crack concrete model. It considers cracking and crushing of the concrete, and when it is combined with a steel material it considers yield of the steel. Compressive strength reduction based on perpendicular tensile strain is accounted for as described in Vecchio and Collins (1986). The model is intended for reinforced concrete and does not account for the tensile strength of concrete. The model does not consider bond slip and dowel action.

Uniaxial Concrete Material Behavior

The uniaxial stress-strain relationship for the concrete material is shown in Figure 1. The material has zero strength in tension. The compression behavior monotonically increases from point O to point U and has constant stress between points U and L. The behavior can be trilinear or elastic-perfectly-plastic. Strength loss from point L to point R is optional. There is constant stress between points R and X, which extends indefinitely, represents residual strength in the material.

In this material model, the unloading and reloading moduli are assumed to be equal to E_0, as shown in Figure 2, and the material unloads linearly to zero stress. The unloading and reloading paths are identical. The model allows for unloading, reloading, and arbitrary cyclic loading, as discussed in a later section.

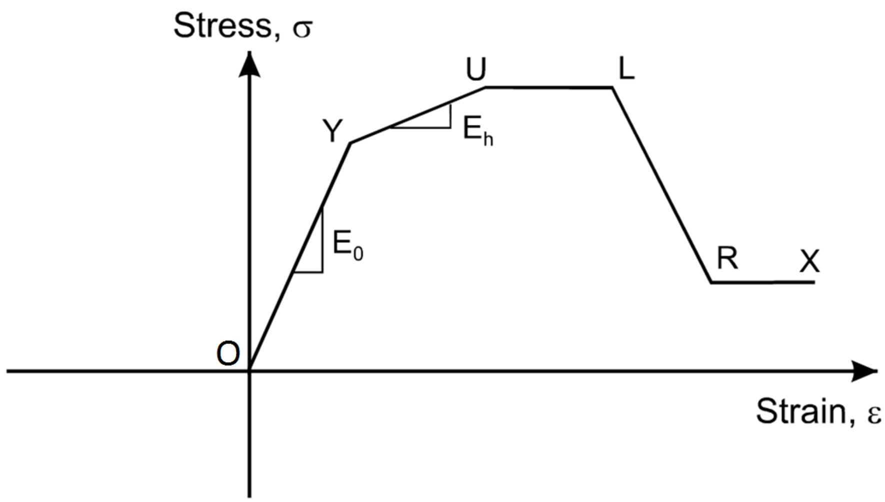

Figure 1 Uniaxial Stress-Strain Relationship

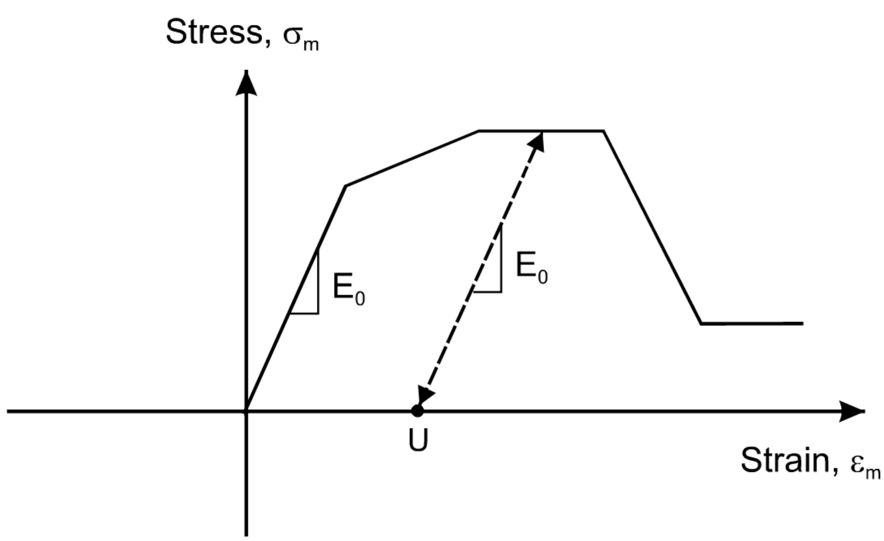

Figure 2 Unloading-Reloading Behavior

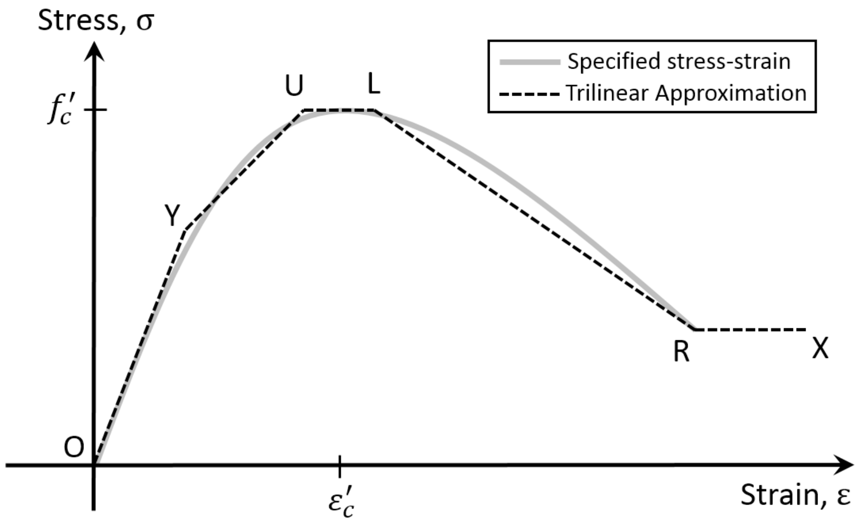

Figure 3 Trilinear Approximation of Concrete Stress-Strain Curve

If an arbitrary material stress-strain curve is specified, a tri-linear approximation will be constructed as shown in Figure 3. In Figure 3, $f_c{}'$ is the peak compression stress and $\varepsilon_c{}'$ is the strain corresponding to the peak compression stress. The approximation is done with the following rules:

1. The initial stiffness E_0 of the tri-linear approximation is equal to the slope between the origin and closest material stress-strain point in compression (the initial stiffness of the user-defined curve).

2. The Y point is determined so that the area under the trilinear stress-strain relationship up to the point of maximum compression stress $f_c{}'$ is identical to that of the user-defined stress-strain curve.

3. The U point is defined by the point of maximum compression stress closest to the origin. The L point is defined by the point of maximum compression stress furthest from the origin. If the U and L points coincide (such as in Figure 3), the U point is placed at the strain value corresponding to $0.98 f_c^{'}$ on the defined stress-strain curve. The L point is placed at the greater of two strain values: (a) the strain corresponding to the maximum compression stress, or (b) 1.05 times the strain at the U point.

4. If the defined material stress-strain curve has strength loss after the point of maximum compression stress, the R point is defined by the last defined point on the material curve.

Initial and Principle Material Axes

Figure 3 shows a wall element, and the stresses at a point in the wall. The initial material axes are fixed relative to the wall element. In general there can be normal and shear stresses in these axes, as shown in Figure 4(b). There are also principal material axes, as shown in Figure 4(c). These axes are parallel to the principal stress directions, and thus are only normal stresses. The key assumption of the Darwin model is that a uniaxial stress-strain relationship can be applied along each of the principal material axes.

Note that although the shear stress is zero in the principal material axes, the shear modulus is not zero. Hence, when a strain increment is applied, the change in shear stress generally will not be zero. During an analysis, the principal stress directions, and hence the principal material axes, can rotate progressively. For this material model, the effective stress-strain relationships in the principal material axes as they rotate are obtained by interpolating between the relationships in the axes of the previous step.

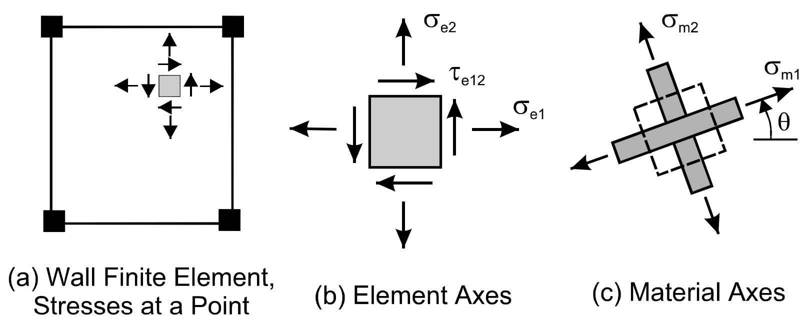

Figure 4 Initial and Principle Material Axes

Initial Elastic Stress-Strain Relationship and Yield Surface for Biaxial Stress

If the material has not yet yielded or cracked (i.e. the material is between the O and Y points as defined in Figure 1), the material has a linear elastic relationship with the initial value of Young's modulus is E_0, and Poisson's ratio is ν, as follows:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_0 & \nu E_0 & 0 \\ \nu E_0 & E_0 & 0 \\ 0 & 0 & E_0 \dfrac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\tau_{12} \end{Bmatrix} \tag{1.1}$$

This equation is independent of the stress and strain directions, and hence it applies in both the initial and principal material axes.

This material model uses a rectangular interaction surface with no explicit stress interaction in the two directions. The effect of biaxial compression stresses on the compression strength of the concrete material is not accounted for. The interaction between stress and tensile strain is discussed in later section.

Post-yield or Cracked Material Behavior

After yield or cracking, the material modulus changes and the Poisson's ratio is neglected. For example, at the Y point in Figure 1 the modulus reduces to E_h, and at the U point it reduces to zero. In general, the stresses, strains and moduli will be different along the two principal directions. Equation 1.1 can be modified for the material nonlinearity as follows:

$$\begin{Bmatrix} d\sigma_{m1} \\ d\sigma_{m2} \\ d\tau_{m12} \end{Bmatrix} = \left[\mathbf{D}_{epm}\right] \begin{Bmatrix} d\varepsilon_{m1} \\ d\varepsilon_{m2} \\ d\tau_{m12} \end{Bmatrix} \tag{1.2a}$$

or

$$d\sigma_m = \mathbf{D}_{epm} d\varepsilon_m \tag{1.2b}$$

where $\mathbf{D}_{epm}$ is the *elastic-plastic constitutive matrix* in principal material axes, given by:

$$\mathbf{D}_{epm} = \frac{1}{1-\nu^2}\begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & G_m \end{bmatrix} \tag{1.3}$$

The shear modulus in the principle material axes, G_m, is specified to maintain coaxiality between the principal stresses and strains.

The corresponding relationship in the initial material axes is obtained by applying the rotation between the initial and principal material axes – a rotation by angle θ, as shown in Figure 4.

Strength Reduction under Perpendicular Tensile Strain

When concrete is subjected to shear stresses, it often cracks in one direction and is in compression in the other direction. Failure in shear may occur when the concrete crushes in compression. Vecchio and Collins (1986) showed that the compression strength of concrete depends on the magnitude of the tensile strain in the perpendicular direction. The effective compression strength of concrete in such situations can be substantially smaller than the original f_c'. Figure 5 shows the relationship between the compression strength and perpendicular tensile strain developed in Vecchio and Collins (1986) and implemented in this material model.

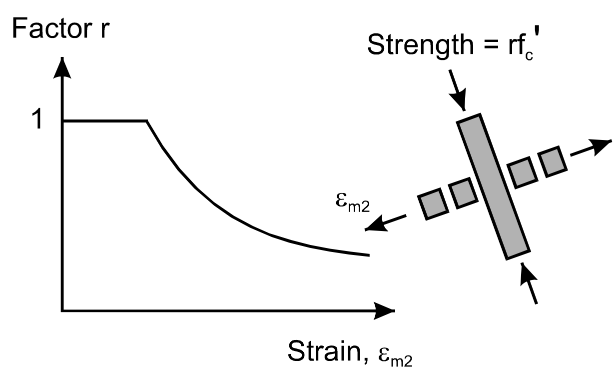

Figure 5 Reduction in Compression Strength Due to Tensile Strain in the Perpendicular Direction

The following equation from Vecchio and Collins (1986) is used for the compression strength reduction factor, r:

$$r = \frac{1}{0.8 - 0.34\dfrac{\varepsilon_m}{\varepsilon_c'}} \leq 1 \tag{1.3}$$

where ε_m is the instantaneous tension strain (positive) in the perpendicular direction and ε_c' is the specified uniaxial crushing strain in compression (negative). The behavior of the material model is as follows:

1. If the concrete is in compression strain along one material axis and tensile strain along the other, the compression strength reduction factor is calculated using Eq. 1.3. The minimum compression strength reduction factor used is based on magnitude of compressive stress, σ, as follows:

$$r_{\min} = \begin{array}{ll} 1.0 & \sigma < 0.2 f_c' \\ 0.25 & \sigma > 0.5 f_c \end{array} \tag{1.4}$$

Linearly interpolated for $0.2 f_c' < \sigma < 0.5 f_c'$

2. The strength in the compression direction may previously have been reduced. If the new reduction factor is smaller than the old one, the new factor is applied. If not, the new factor is ignored.
3. If the new factor is applied, the stress-strain relationship is modified as indicated in Figure 6. Note that the moduli E_0 and E_h do not change.

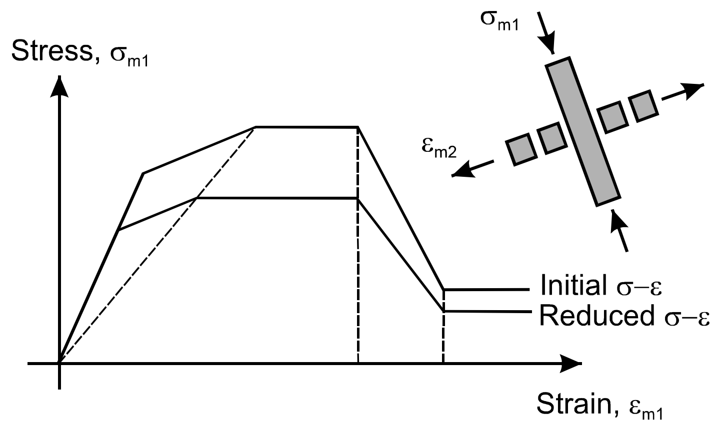

Figure 6 Change in Stress-Strain Relationship to Account for Strength Reduction

Important Numerical Considerations

General guidelines for working with nonlinear analysis are addressed in Topic "Important Considerations" in Chapter 23 of the *CSI Analysis Reference Manual*. Specific numerical considerations for this material model are as follows:

1. Compared to the directional material models, the Modified Darwin-Pecknold model has a higher degree of nonlinearity and may require smaller time steps to converge.
2. This material is used in shell elements, which use a two-by-two numerical integration formulation in the plane. Some refinement of the mesh may be needed to capture varying nonlinear behavior. However care should be taken not to over-refine because localized failure may occur for very small mesh sizes.
3. As an alternative to nonlinear static analysis, using a nonlinear dynamic analysis with slowly applied excitation may result in better convergence behavior. This applies especially to cases where significant crushing or loss of strength is expected.

4. Setting the Poisson’s ratio equal to zero in the material properties may improve convergence in some cases because this decreases the initial coupling between the two material axes.

References

Darwin, D. and Pecknold, D.A.W., "Inelastic Model for Cyclic Biaxial Loading of Reinforced Concrete", University of Illinois, July 1974.

Vecchio, F.J. and Collins, M.P., "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear", Journal of the ACI, Paper No. 83-22, March-April 1986.